Design of Restructurable Flight Control Systems Using Feedback Linearization

Yoshimasa Ochi* and Kimio Kanai†
National Defense Academy, Yokosuka, 239, Japan

In this paper, a design method is presented for restructurable flight control systems based on the feedback linearization method. Failures are identified indirectly by estimating parameters of the nonlinear aircraft model using the recursive least square algorithm. The aircraft is assumed to have many control surfaces that can be driven independently. In the design, actuator dynamics are taken into account and the control distributor, which reduces real inputs to generic inputs, is used. The imaginary actuators for generic inputs are introduced to generate input signals used in parameter identification. Pitch and roll angles are controlled indirectly by controlling pitch and roll rates, respectively, which is an approximate way but makes the control system simpler than applying the feedback linearization method straight to the control of the angles. To evaluate the performance of the restructurable flight control system, two failure cases are simulated on the six-degree-of-freedom nonlinear aircraft model.

I. Introduction

A RESTRUCTURABLE flight control system (RFCS) typically consists of a failure detection and identification (FDI) module, a redesign module, which modifies the controller by, for example, changing control parameters, and a baseline flight control system.[1] What should be done first when failures occur in aircraft dynamics, such as loss of a surface, is to stabilize and settle the aircraft motion at a new trim point, following quick and accurate FDI.

In most of the studies on RFCS so far, the baseline controller is designed based on the linear control theory, where it is important to find a new trim (equilibrium) point.[2] Linear controllers generally work well for small variations of state or control variables. However, when a failed aircraft has coupled motions or motions in which nonlinearity cannot be neglected, the control law will lose its effectiveness. Moreover, exact identification of a linear system will be difficult in such a situation. Particularly, it will be almost impossible to find a trim point.

From this viewpoint, the RFCS based on the feedback linearization,[3,4] which is an established design method for nonlinear dynamic systems and has been applied to flight control,[5-7] is proposed in this paper. The first feature of this RFCS is that it is an adaptive-type controller. Failures considered in this paper are those whose effects appear as parameter change in the aircraft model. They are identified indirectly by estimating the parameters of the equations of motion of the aircraft discretely using a method such as the recursive least square algorithm, and the estimated parameters are used to compute new control parameters. The second feature is that the control distributor (CD)[8,9] and generic inputs are incorporated. It is desirable to have many control effectors that can be driven independently to accommodate failures. In fact, an aircraft such as the control reconfigurable combat aircraft (CRCA)[10] has 17 surfaces. However, in this design, the number of inputs must be at least equal to that of outputs to be controlled. A reason why CD is introduced is to reduce the real input vector to the generic input vector, which can be defined to have the same number of elements as the output vector. Unlike in Refs. 9 or 10, when failures occur, it is not the control distributor but the control law that is changed in this RFCS. In addition, in an aircraft having many controls, the number of parameters to be identified is large and independence of the input signal vectors can be small in longitudinal- and lateral-directional decoupled motions, which generally makes parameter identification difficult. By using generic inputs in identification, the problems will be alleviated. However, the generic inputs must be modified when the real inputs are saturated so that parameters can be identified correctly. This is also referred to in Sec. III.

In the design of flight control systems, neglecting actuator dynamics sometimes greatly degrades the nominal performance guaranteed. In this RFCS, actuator dynamics are also taken into account. In order to use generic inputs in parameter identification, an actuator for a generic input, which is called an imaginary actuator in this paper, is introduced.

The next section describes the baseline control system, and the description of its adaptive version follows in Sec. III. In Sec. IV, the performance and the characteristics of the RFCS are investigated by computer simulation. Finally, the summary and conclusions are presented in Sec. V.

II. Baseline Control System

State and Output Equations

It is assumed that the aircraft dynamics and the variables to be controlled are represented by the following state and output equations, respectively,

$$\dot{X} = A(X) + B(X)U \tag{1}$$

$$Y = CX \tag{2}$$

where $X \in R^n$, $U \in R^m$, and $Y \in R^l$ are state, control, and output vectors, respectively, and $A(X)$ $(\in R^n)$ and $B(X)$ $(\in R^{n \times m})$ have the form of linear combination of constant parameters and known functions of the state variables. $C(\in R^{m \times n})$ is a constant matrix.

State Equations of Actuators

Actuator dynamics are assumed to be described by

$$\dot{U} = \Lambda(-U + U_C) \tag{3}$$

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*Research Assistant, Aerospace Engineering, 1-10-20 Hashirimizu.
†Professor, Aerospace Engineering, 1-10-20 Hashirimizu. Member AIAA.

where $\Lambda = \mathrm{diag}(1/T_i)$, $(i = 1, \ldots, m)$, T_i is the time constant of each actuator, and U_C is an input vector to the actuators.

Control Distributor and Generic Input

Since the aircraft considered here has many surfaces, it is reasonable to assume that $l < m$. In order to reduce the number of inputs to that of outputs, the control distributor and generic inputs are introduced.[8,9]

A generic input vector $U_G(\in R^{m'})$ is defined by

$$U = PU_G \tag{4}$$

where $P(\in R^{m\times m'})$ is called a control distributor matrix and assumed to have full rank and to be constant here, and m' is given to be equal to l. Using P and defining $U_{CD}(\in R^m)$ as the outputs of CD, a generic input vector for actuator inputs, $U_{CG}(\in R^{m'})$, is also defined by

$$U_{CD} = PU_{CG} \tag{5}$$

where the relation between U_{CD} and U_C will be given by Eq. (23) later.

Control Law

The control law is derived by the feedback linearization method[3,4] as follows.

Differentiating both sides of Eq. (2), using Eqs. (1) and (4), yields

$$\dot{Y} = A'(X) + B_G'(X)U_G \tag{6}$$

where $A'(X)(\in R^l) = CA(X)$ and $B_G'(X)(\in R^{l\times l}) = CB(X)P$.

Defining $Y' = \dot{Y}$, differentiating Eq. (6) and using Eq. (3), yields

$$\dot{Y}' = A_X'(X, U_G) + B_{XG}'(X)U_{CG} \tag{7a}$$

where

$$A_X'(X,U_G) = C\left\{\left[\frac{\partial A(X)}{\partial X^T} + \sum_{i=1}^{l}\frac{\partial B_{Gi}(X)}{\partial X^T}U_{Gi}\right]\right.$$
$$\left.\times[A(X) + B_G(X)U_G] - B_G(X)\Lambda PU_G\right\} \tag{7b}$$

$$B_{XG}'(X) = CB_G(X)\Lambda P \tag{7c}$$

where $U_G = [U_{G1}, \ldots, U_{Gl}]^T$, $B_G(X) = B(X)P = [B_{G1}(X), \ldots, B_{Gl}(X)]$, and $B_{Gi}(X)\in R^n$. From Eqs. (7), the control law for generic inputs by which Y tracks the reference outputs Y^* is given by

$$U_{CG} = B_{XG}'(X)^{-1}[-A_X'(X,U_G) + G'z + \ddot{Y}^*] \tag{8}$$

where $z = [y^T, y'^T]^T$, $y' = \dot{Y} - \dot{Y}^*$, $G'\in R^{l\times 2l}$ and $B_{XG}'(X)$ is assumed to be nonsingular.

Substituting Eq. (8) into Eqs. (7) and using the definition of y', the augmented system is described by

$$\dot{z} = Ez \tag{9}$$

where

$$E = \begin{bmatrix} 0_{ll} & I_l \\ & G' \end{bmatrix} \in R^{2l\times 2l}$$

$I_l(\in R^{l\times l})$ is an identity matrix, and $0_{ll}(\in R^{l\times l})$ is a zero matrix. If G', which makes E a Hurwitz matrix, is determined, the closed-loop system for the outputs is stable, i.e., $y\to 0$ or $Y\to Y^*$ as $t\to\infty$.

When $B_{XG}'(X)$ is singular, the outputs must be differentiated and the state equations substituted into $\dot{X}$ until a nonsingular coefficient matrix of U_G appears.[3,4] However, there is a case where the outputs can be controlled without differentiating the output equations.

In general, let us consider the following output differential equations for $Y_1(\in R)$ and $Y_2(\in R)$, and let Y_1 be the original output to be controlled

$$\dot{Y}_1 = A_1'(X) \tag{10}$$

$$\dot{Y}_2 = A_2'(X) + B_{G2}'(X)U_G \tag{11}$$

where $U_G\in R$, $[A_1'(X), A_2'(X)]^T = A'(X)$ and $[0, B_{G2}'(X)]^T = B_G'(X)$ in Eq. (6). Since there is no control variable in Eq. (10), this is a case where $B_{XG}'(X)$ is singular. Suppose that Eq. (10) can be solved for Y_2 as

$$Y_2 = a_1'(X, \dot{Y}_1) \tag{12}$$

Let us give the reference of Y_2 as

$$Y_2^* = a_1'[X, G_{Y1}(Y_1 - Y_1^*) + \dot{Y}_1^*] \tag{13}$$

where G_{Y1} is a proper negative constant. If $B_{G2}'(X)$ is nonsingular, it is possible that Y_2 tracks Y_2^* by the same control law as Eq. (8). When $Y_2\to Y_2^*$ is attained, Y_2 in Eq. (10) can be approximately replaced by Y_2^* in Eq. (13). Substituting Y_2^* into Y_2 in Eq. (10) yields

$$\dot{Y}_1 - \dot{Y}_1^* = G_{Y1}(Y_1 - Y_1^*) \tag{14}$$

Since $G_{Y1} < 0$, Y_1 converges to Y_1^*.

In this method, there is no need to compute partial derivatives of $A_1'(X)$ as is required when the feedback linearization method is applied straight to the control of Y_1 though the case where it is effective is limited to where Y_2 can be obtained, as Eq. (12). When actuator dynamics are taken into account, partial derivatives must be computed to obtain the control law, as shown in Eq. (7b) but unless this method is employed to control Y_1, one is required to compute the second partial derivatives, which makes the design more complicated.

III. Adaptive Control System

When the parameters of $A_X'(X,U_G)$ and $B_{XG}'(X)$ change from the nominal ones due to failures, the control law, Eq. (8), is replaced by

$$U_{CG} = \hat{B}_{XG}'(X)^{-1}[-\hat{A}_X'(X,U_G) + G'z + \ddot{Y}^*] \tag{15}$$

where the parameters of $\hat{A}_X'(X,U_G)$ and $\hat{B}_{XG}'(X)$ are the estimates of $A_X'(X,U_G)$ and $B_{XG}'(X)$, respectively.

Parameter Identification

In this RFCS, while the control law is derived based on the continuous time control system so that the parameters of the continuous system must be identified, the parameters are updated discretely because the discrete time identification is generally superior to the continuous one in convergence. It results in what is called a hybrid adaptive controller. Introducing state variable filters, the signals used in the identification are generated through the filters from states, outputs, inputs, and functions of them so as not to use the differential signals of the outputs.

The filter used here is $1/(s+\lambda)$, where s is a Laplace or differential operator and λ is a proper positive constant.

Multiplying both sides of Eq. (6) by $1/(s+\lambda)$ yields

$$Y - \lambda Y_f = A_f'(X) + B_{Gf}'(X, U_G) \tag{16}$$

where $Y_f = Y/(s+\lambda)$, $A_f'(X) = A'(X)/(s+\lambda)$, and $B_{Gf}'(X,U_G) = \{B_G'(X)U_G\}/(s+\lambda)$. Since $A_f'(X)$ and $B_{Gf}'(X,U_G)$ also have the form of linear combination of the constant parameters and the functions of X and U_G, which are given by filtering the known functions, the equation for the ith element of Eq. (16) can be rewritten in a parametric form as

$$\bar{Y}_i = \xi_i^T \eta_i, \qquad i = 1, \ldots, m \tag{17}$$

where $\bar{Y}_i = Y_i - \lambda Y_{fi}$, ξ_i is composed of the filtered functions, and η_i is composed of the constant parameters.

In Eq. (17), since $\bar{Y}_i$ and ξ_i are accessible, η_i can be identified using the recursive least square method. Thus, the parameters of $A'(X)$ and $B_G'(X)$ are identified. However, in order to find control inputs, the parameters of $A_X'(X, U_G)$ and $B_{XG}'(X)$ in Eq. (15) must be obtained. When one tries to identify all of the parameters of Eq. (15), the number of parameters to be identified increases remarkably, compared with that of the RFCS designed neglecting actuator dynamics, where the parameters of Eq. (6) have only to be identified. This is the reason why the approach of identifying the parameters of $A'(X)$ and $B_G'(X)$ based on Eq. (6) and computing the parameters of $A_X'(X, U_G)$ and $B_{XG}'(X)$ is taken.

Let us show the convergence characteristics of the identification error ϵ and the tracking error y. ϵ is defined by

$$\epsilon = Y - \lambda Y_f - \hat{A}_f'(X) - \hat{B}_{Gf}'(X, U_G) \tag{18}$$

where $\hat{A}_f'(X)$ and $\hat{B}_{Gf}'(X, U_G)$ are defined by replacing the constant parameters of $A_f'(X)$ and $B_{Gf}'(X, U_G)$ with their estimates, respectively. Defining ϵ' as

$$\epsilon' = \ddot{Y} - \hat{A}_X'(X, U_G) - \hat{B}_{XG}'(X) U_{CG} \tag{19}$$

the following equation is obtained.

$$z = (sI_{2l} - E)^{-1} [0_l^T \epsilon'^T]^T \tag{20}$$

where $0_l = [0, \ldots, 0]^T \in R^l$. As the parameters converge to the true values, from the definition of ϵ',

$$\epsilon' \to s[\dot{Y} - \hat{A}'(X) - \hat{B}_G'(X) U_G] = s(s + \lambda)\epsilon \tag{21}$$

Equations (20) and (21) indicate that $z \to 0$, i.e., $y \to 0$, if convergence of the estimates to the true values are attained.

CD plays an important role in parameter identification. One advantage of using CD is that the number of parameters of $\hat{B}_G'(X)$ is less than that of the coefficient matrix of U. In fact, in the aircraft model used in simulation, the number of parameters with CD is 139, as compared with 187 without CD. Another advantage is that U_G is expected to have more independence than U in longitudinal- and lateral-directional decoupled motion if P is properly given.

When CD is used, the real inputs to the aircraft U are obtained as the actuator outputs. However, the generic inputs U_G, which are used to identify the parameters of $A'(X)$ and $B_G'(X)$, cannot be found from Eq. (4) because $m' < m$. In order to find U_G, which satisfies Eq. (4), an imaginary actuator is introduced.

Suppose that all of the actuators have the same time constant, i.e., $\Lambda = \text{diag}(1/T)$. Since P is given to have full rank, then Eq. (3) becomes

$$\dot{U}_G = (-U_G + U_{CG})/T \tag{22}$$

This equation represents the imaginary actuator dynamics.

In the case where the time constants of the actuators are different, it is necessary to feed U back to U_C so that all of the eigenvalues of the closed-loop system concerning the actuators can be $-1/T$. This is achieved by using the following actuator inputs,

$$U_C = (I_m - \Lambda^{-1}/T)U + (\Lambda^{-1}/T)U_{CD} \tag{23}$$

where $I_m \in R^{m \times m}$ is an identity matrix. Regarding U_{CD} as new actuator inputs and the closed-loop system as new actuators, their dynamics are also described by Eq. (22). If T is given to be equal to or greater than the largest time constant, U_C does not violate the displacement constraints of the surfaces. Therefore, the constraints have only to be imposed on U_{CD}. The block diagram of the RFCS is shown in Fig. 1.

Input Saturation and Modification of U_G

Available inputs are bounded in practice, and when they are saturated at their boundaries of the feasible control regions, the inputs U_{CD}, generated by the controller, are different from the real saturated inputs U. For simplicity, let us neglect actuator dynamics. If U is not saturated, then $U = U_{CD}$, but if it is saturated, $U \neq U_{CD}$ in general, hence, $U \neq PU_G$. Thus, since the generic inputs do not necessarily correspond to the real saturated inputs, in order to identify the unknown parameters correctly using the generic inputs, U_G, which satisfies Eq. (4), must be found. However, since $m > m'$, it is generally impossible to find such U_G from Eq. (4) by the pseudoinverse of P. The following is a solution to this problem.

Suppose that the constraint represented by the following inequality is imposed on U_i, the ith element of U,

$$-U_{i\max} \le U_i \le U_{i\max}, \qquad i = 1, \ldots, m \tag{24}$$

Let us define $M_{ri} = |U_{CDi}/U_{i\max}|$, where U_{CDi} is the ith element of U_{CD}, and determine M_r,

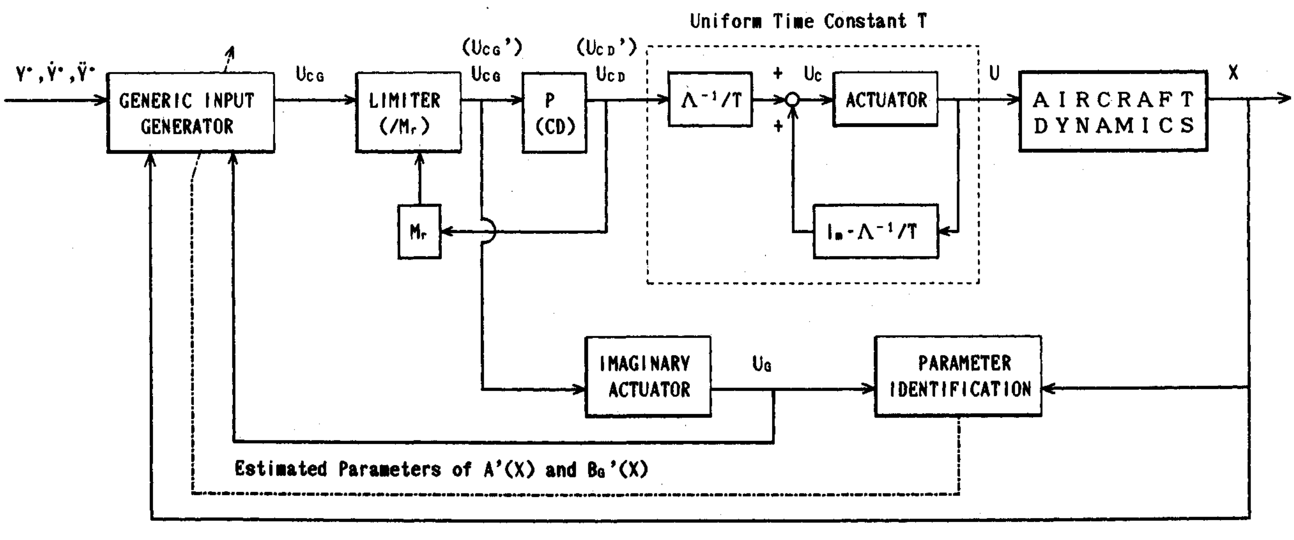


Fig. 1 Block diagram of the RFCS.

$$M_r = \max(1, M_{r1}, \ldots, M_{rm}) \tag{25}$$

Dividing U_G by M_r given by Eq. (25) prevents any element of $U' = PU_G/M_r$ from violating the constraints. The modified real inputs U', which are actually applied to the aircraft, satisfy the relation $U' = PU_G'$, where $U_G' = U_G/M_r$. When actuator dynamics are considered, violation of the constraints by U_{CD} does not always mean that by U. However, for safety, this modification is applied to the actuator inputs U_{CD}, so that U and U_G always satisfy Eq. (4).

Summary of the Design Procedure

The final design procedure is summarized as follows.

1) Choose the outputs to be controlled and give the reference outputs.

2) Determine the dimension of U_G and define the control distributor matrix P.

3) If actuator time constants are different, compensate the dynamics by using actuator inputs given by Eq. (23) so that all of the time constants of the closed-loop system are the largest one of the real actuators.

4) Define the imaginary actuator, Eq. (22), whose time constant is given by the largest one of the real actuators.

5) Make a parameter identifier based on the model of Eq. (16), where the parameters of $A'(X)$ and $B_G'(X)$ are estimated discretely by the recursive least square algorithm using U_G and X.

6) Control inputs to the real actuators are given by Eqs. (23), (5), and (15). The unknown parameters of the control law are computed from Eqs. (7b) and (7c) using the estimated parameters of $A'(X)$ and $B_G'(X)$.

7) If any element of U_{CD} violates the constraints, it is modified by $U_{CD}' = PU_{CG}'$, where $U_{CG}' = U_{CG}/M_r$ and M_r is determined by Eq. (25) on line.

Finally, in this section, let us mention the problems of the design. The specific one to this RFCS is that the entire control system including the parameter identifier is so large and complicated that difficulties may arise in implementation, such as performance degradation due to discretizing the control law and computation delay or requiring high performance computers to compensate for it, etc. A generic problem is robustness to modeling errors of the aircraft or actuator dynamics, measurement noise, unstational turbulences, and sensor failures, etc. These are important problems, but they are beyond the scope of this effort.

IV. Simulation

The performance and the characteristics of the RFCS were evaluated through computer simulation using 6-DOF nonlinear equations of motion of a fighter aircraft having seven control surfaces. Four cases including no failure case are simulated, and convergence properties of the errors and the effects of the generic input modification on real inputs are also examined.

Mathematical Model of Aircraft

The aircraft dynamics are described by the 6-DOF nonlinear equations of motion as follows[11]:

$$\dot{u} = -g\sin\theta + vr - wq + (\rho V^2 S/2M_a)(C_X + C_{X\delta}^T\delta) + T_h/M_a \tag{26}$$

$$\dot{w} = g\cos\theta\cos\phi + uq - vp + (\rho V^2 S/2M_a)(C_Z + C_{Z\delta}^T\delta) \tag{27}$$

$$\dot{q} = [(I_Z - I_X)/I_Y]pr + (\rho V^2 Sc/2I_Y)(C_m + C_{m\delta}^T\delta) + (\rho V S c^2/4I_Y)C_{mq}q \tag{28}$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \tag{29}$$

$$\dot{v} = g\cos\theta\sin\phi + wp - ur + (\rho V^2 S/2M_a) \times (C_Y + C_{Y\delta}^T\delta + C_{Y\delta r}\delta_r) + (\rho V S b/4M_a)(C_{Yp}p + C_{Yr}r) \tag{30}$$

$$\dot{r} = [(I_X - I_Y)/I_Z]pq + (\rho V^2 Sb/2I_Z)(C_n + C_{n\delta}^T\delta + C_{n\delta r}\delta_r) + (\rho V S b^2/4IZ)(C_{np}p + C_{nr}r) \tag{31}$$

$$\dot{p} = [(I_Y - I_Z)/I_X]qr + (\rho V^2 Sb/2I_X)(C_l + C_{l\delta}^T\delta + C_{l\delta r}\delta_r) + (\rho V S b^2/4I_X)(C_{lp}p + C_{lr}r) \tag{32}$$

$$\dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \tag{33}$$

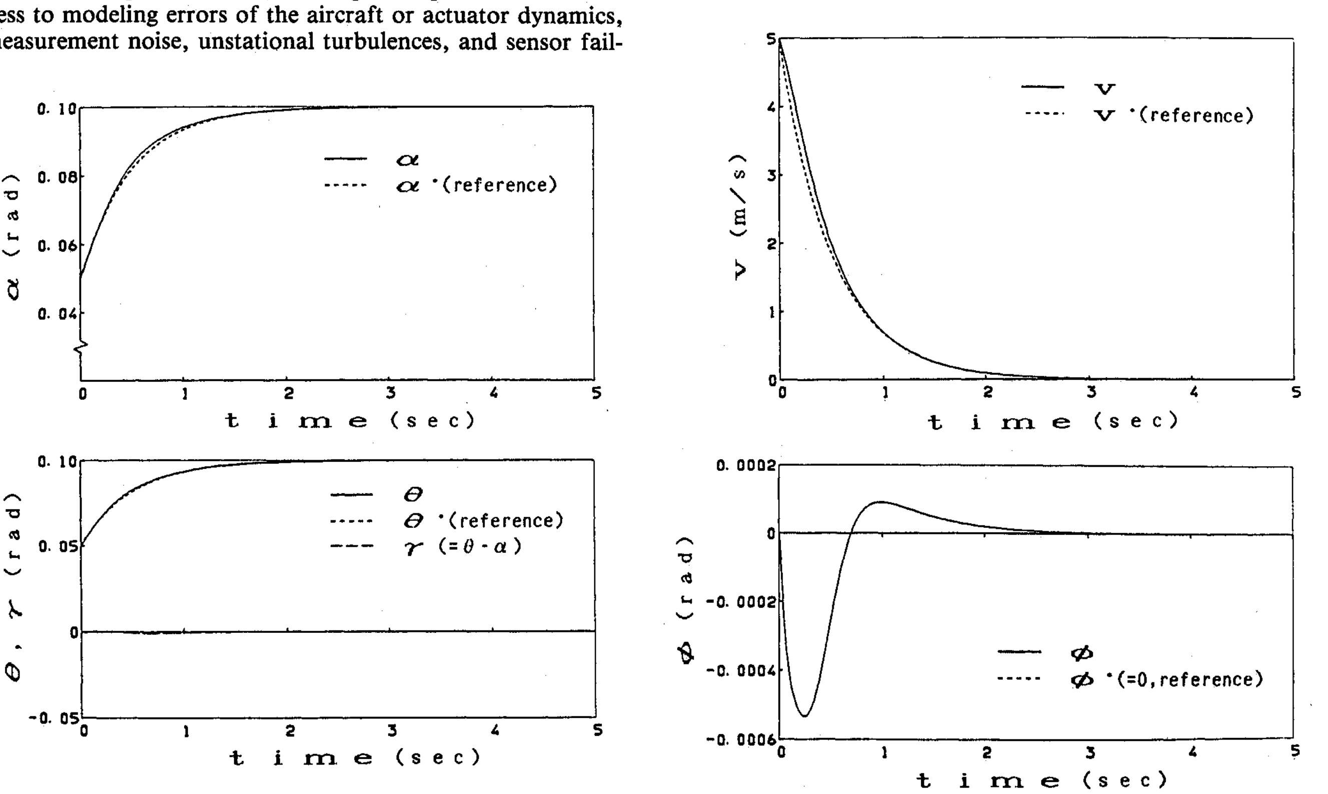
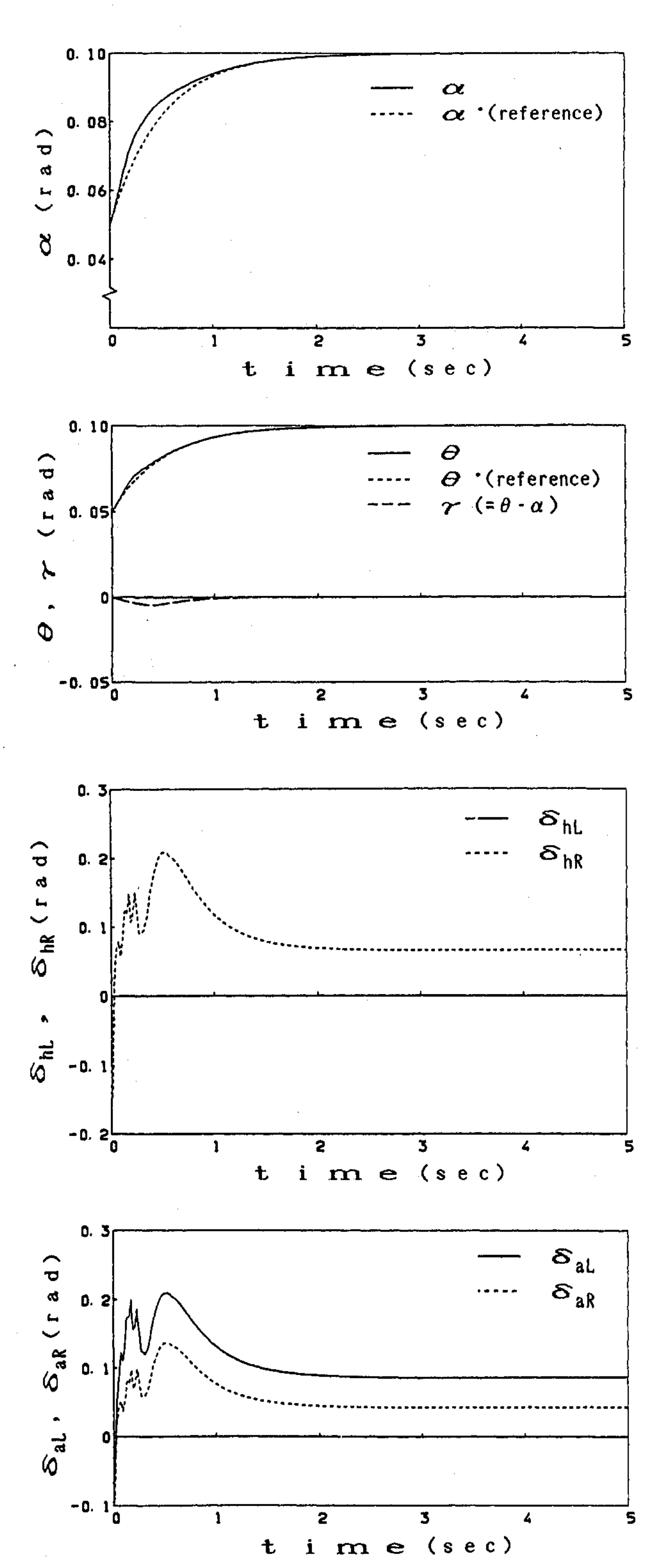


Fig. 2 Output responses of the normal case.

Actuator dynamics are assumed to be

$$\dot{\boldsymbol{\delta}} = \Lambda(-\bar{\boldsymbol{\delta}} + \bar{\boldsymbol{\delta}}_c) \tag{34}$$

where $\boldsymbol{\delta} = [\delta_{hL}, \delta_{hR}, \delta_{aL}, \delta_{aR}, \delta_{cL}, \delta_{cR}]^T$ is the surface deflection vector, $\boldsymbol{\delta}_c = [\delta_{hLc}, \delta_{hRc}, \delta_{aLc}, \delta_{aRc}, \delta_{cLc}, \delta_{cRc}]^T$ is the command input vector to the actuators of the surfaces $\bar{\boldsymbol{\delta}} = [\boldsymbol{\delta}, \delta_r]^T$ and $\bar{\boldsymbol{\delta}}_c = [\boldsymbol{\delta}_c, \delta_{rc}]^T$, and $\boldsymbol{C}_{i\delta} = [C_{i\delta hL}, C_{i\delta hR}, C_{i\delta aL}, C_{i\delta aR}, C_{i\delta cL}, C_{i\delta cR}]^T$ $(i = X,Z,m,Y,n,l)$ are the nondimensional coefficients, which are functions of angle of attack α or side slip angle β. The subscripts indicate that X,Y,Z are values with respect to forward, sideward, and downward body axes, respectively, and l,m,n are values with respect to roll, pitch, and yaw, respectively. As for the surfaces, the subscripts indicate that h is the horizontal tail, a the aileron, c the canard, r the rudder, L the left surface, and R the right surface. Other symbols are the following: u is the speed along the X axis (m/s), w the speed along the Z axis (m/s), q the pitch rate (rad/s), θ the pitch angle (rad), v the speed along the Y axis (m/s), r the yaw rate (rad/s), p the roll rate (rad/s), ϕ the roll



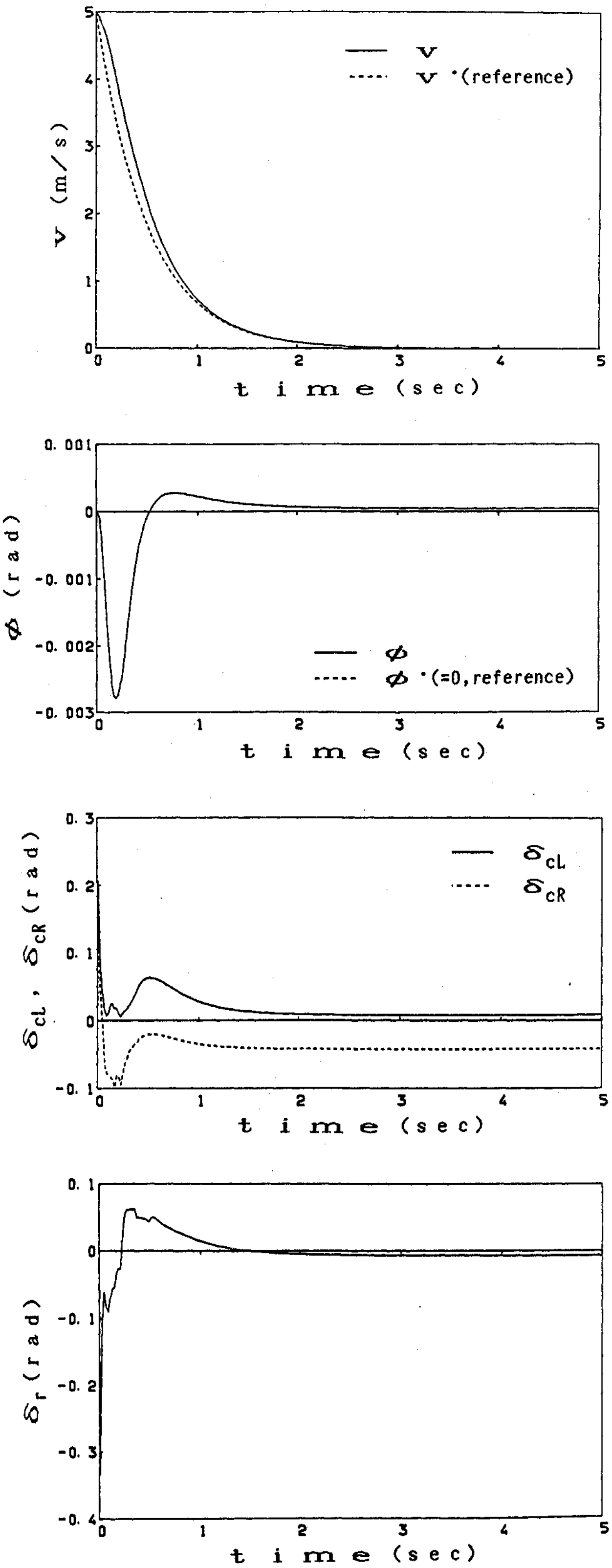


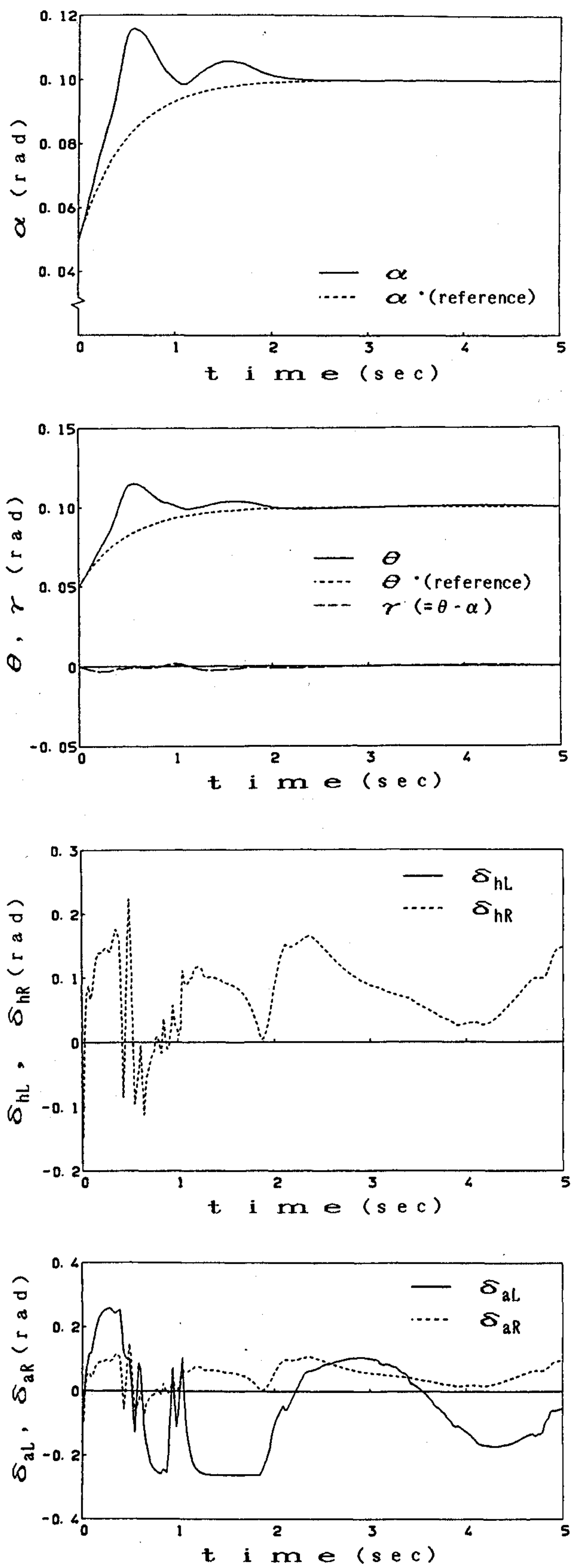
Fig. 3 Output and control responses of the failure case a.

angle (rad), g the acceleration of gravity, ρ the density of atmosphere, V the air speed, S the wing area, M_a the mass, T_h the thrust, c the mean aerodynamic chord length, b the wing span, and I_X, I_Y, I_Z the moments of inertia about X, Y, Z axes, respectively.

In the design of the RFCS, α (rad) is used instead of w as a state variable. State variables are $X = [u, \alpha, q, \theta, v, r, p, \phi, \bar{\delta}^T]^T$, and control variables are $U_c = [\bar{\delta}_c^T, T_h]^T$, where T_h is assumed to be constant in the simulation.

Numerical data of the aircraft configuration are $M_a = 22{,}695$ kg, $S = 48.77$ m^2, $b = 19.2$ m, $c = 2.76$ m, $I_X = 67{,}790$ kgm^2, $I_Y = 427{,}348$ kgm^2, $I_Z = 476{,}564$ kgm^2, $I_{XZ} = 0$ kgm^2, and $g = 9.8$ m/s^2. Nominal flight conditions are altitude = 7600 m, $\rho = 0.5495$ kg/m^3, $V = 220$ m/s (0.71M).

Since, in Ref. 11, nondimensional forces, moments (C_X, C_m, etc.), and derivatives (C_{mq}, $C_{x\delta hl}$, etc.) are given by a table as functions of α and β, they are approximated using the data



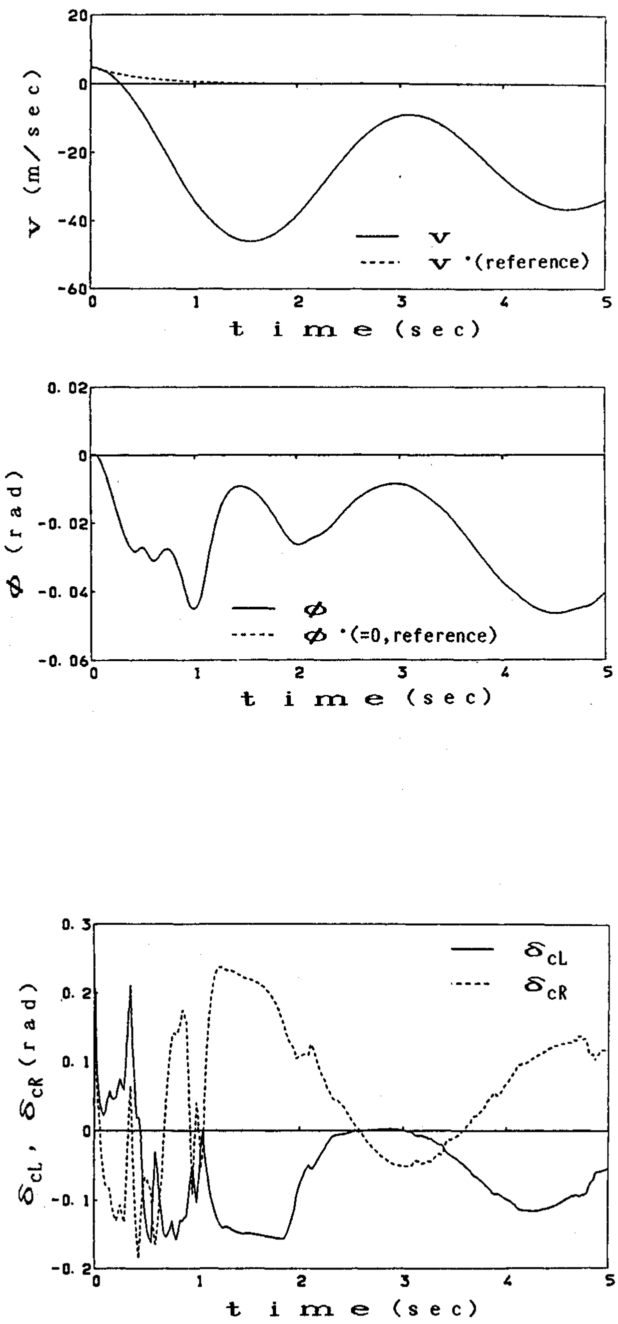


Fig. 4 Output and control responses of the failure case b.

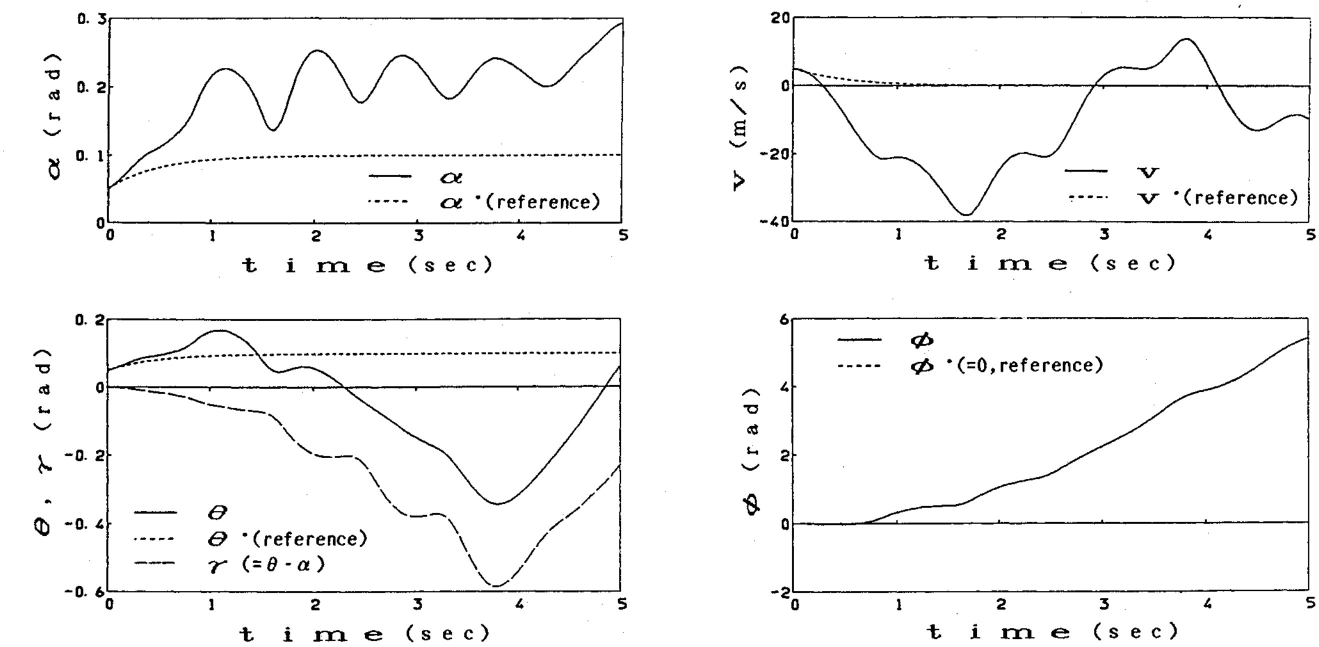


Fig. 5 Output responses of the failure case b without restructure.

for $-10 \le \alpha \le 50$ deg and $-20 \le \beta \le 20$ deg by the polynomials of α and β for C_m, C_Y, C_n, and C_l and by the polynomials of α for the other coefficients, where the polynomials are eighth-order as to α and fourth-order as to β. The state equations of identification model are the same as Eqs. (26-28) and (30-32), and the nondimensional coefficients are modeled by the similar polynominals of α and β to those used in the aircraft model, except that the orders are second as to both α and β.

Let us choose P and U_G as,

$$P = \begin{bmatrix} P_{h1} & 0 & P_{a1} & 0 \\ P_{h1} & 0 & -P_{a1} & 0 \\ P_{h2} & 0 & P_{a2} & 0 \\ P_{h2} & 0 & -P_{a2} & 0 \\ 0 & P_{hc} & P_{ac} & 0 \\ 0 & P_{hc} & -P_{ac} & 0 \\ 0 & 0 & 0 & P_r \end{bmatrix}$$

$$U_G = [\delta_{Gh1}, \delta_{Gh2}, \delta_{Ga}, \delta_{Gr}]^T$$

where, since $\delta_r = \delta_{Gr}$ and δ_{Gr} is independent of other real inputs, the generic input modification is not applied to δ_{Gr}.

The output Y and the reference Y^* are chosen as $Y = [\alpha, \theta, v, \phi]^T$ and $Y^* = [\alpha^*, \theta^*, v^*, \phi^*]^T$, respectively.

For the outputs, since there is no input in Eqs. (29) and (33) so that $B_G'(X)$ is singular, θ and ϕ are controlled by the method represented by Eqs. (10-14), where $Y_1 = \theta$ or ϕ, $Y_2 = q$ or p, and the references Y_2^* are given by

$$q^* = \{r \sin\phi + G_q(\theta - \theta^*) + \dot{\theta}^*\}/\cos\phi \qquad (35)$$

$$p^* = -(q \sin\phi + r \cos\phi)\tan\theta + G_p(\phi - \phi^*) + \dot{\phi}^* \qquad (36)$$

respectively, where G_q and G_p are negative constants. From Eqs. (29) and (35),

$$\dot{\theta} - \dot{\theta}^* = (q - q^*)\cos\phi/(s - G_q) \qquad (37)$$

is obtained, and from Eqs. (33) and (36),

$$\dot{\phi} - \dot{\phi}^* = (p - p^*)/(s - G_p) \qquad (38)$$

where $s = d/dt$. Since $G_q < 0(G_p < 0)$, and $q \to q^*(p \to p^*)$ is guaranteed, it will be attained that $\theta \to \theta^*(\phi \to \phi^*)$.

In Eq. (15), $\dot{q}^*$ and $\ddot{q}^*$ and $\dot{p}^*$ and $\ddot{p}^*$ are also needed. Here, using Y_1 and Y_2 defined earlier, they are given by $\dot{Y}_2^* = G_{Y2}(Y_2 - \dot{Y}_1^*)$ and $\ddot{Y}_2^* = G_{Y2}(\dot{Y}_2 - \ddot{Y}_1^*)$, where $G_{Y2} = G_q$ or G_p.

The parameters or the conditions in the simulation are as follows. Displacement limits for the surfaces, $\delta_{i\max}[= U_{i\max}$ in Eq. (24)] are 0.4 rad for $i = hL, hR$; 0.2618 rad for $i = aL, aR$; 0.3 rad for $i = cL, cR$; 0.5236 rad for $i = r$. Thrust is $T_h = 0.4463 \times 10^5$ N. Sampling period of parameter identification is 0.05 s. Interval of updating control parameters is 0.05 s. Initial conditions are $X(0) = [230, 0.05, 0.05, 0.05, 5.0, 0, 0, 0]^T$. In Eq. (8), $G' = [G_1' \; G_2']$, where $G_1' = \mathrm{diag}\{-25, -100, -25, -100\}$ and $G_2' = \mathrm{diag}\{-10, -20, -10, -20\}$. Filter is $1/(s + 30)$. Initial estimates are nominal parameters of the normal aircraft at the trim point $X_{tr} = [220, 0.1, 0, 0.1, 0, 0, 0, 0]^T$, $U_{tr} = [-0.03046, -0.03046, -0.01994, -0.01994, -0.003592, -0.003592, 0.0, 0.4436 \times 10^5]^T$. Control distributor matrix is $P_{h1} = 0.4$, $P_{a1} = 0.4$, $P_{h2} = 0.2618$, $P_{a2} = 0.2618$, $P_{hc} = -0.3$, $P_{ac} = 0.3$, $P_r = 0.5236$. Actuator time constants are $\Lambda^{-1} = \mathrm{diag}\{0.05, 0.05, 0.04, 0.04, 0.033, 0.033, 0.05\}$ s. Eigenvalues of the linearized system about the nominal trim point are longitudinal $\{-0.00547 \pm 0.04549j, -0.5828 \pm 2.275j\}$, lateral directional $\{-0.2703 \pm 2.166j, -1.033, -0.01393\}[j = (-1)^{1/2}]$. Reference outputs are given by $Y^*(t) = Y^* + [Y(0) - Y^*]\exp(at)$, where $a = -2$, $Y^* = [0.1, 0.1, 0, 0]^T$.

Simulation Results

Under these conditions, time responses of the outputs and the inputs were computed for four cases: 1) no failure case; 2) failure case a, i.e., the left horizontal tail is stuck at -0.2 rad and at the same time the effectiveness of the rudder reduces by 50%; 3) failure case b, i.e., the left horizontal tail and the rudder are stuck at -0.2 rad and at the same time the effectiveness of the left aileron and the right canard reduces by 50%; and 4) failure case b without restructure, i.e., control parameters are not modified. The failures occur at $t = 0$ s. For each case, the simulation results are as follows.

1) Figures 2 show the time responses of the outputs. The tracking errors are acceptably small and disappear in steady state, which indicates that the nominal control system works well.

2) As seen from the output responses shown by Figs. 3, the tracking performance is a little degraded, compared with that

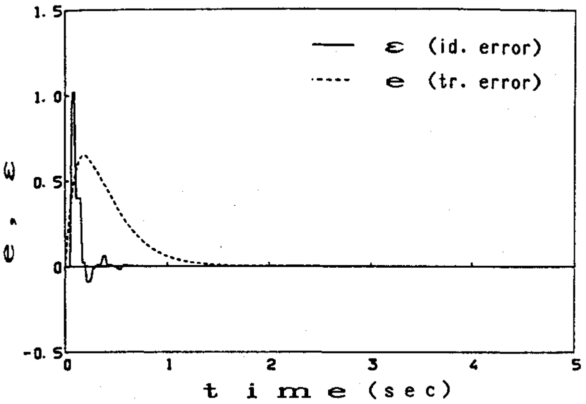


Fig. 6 Time histories of errors for side speed in the failure case a.

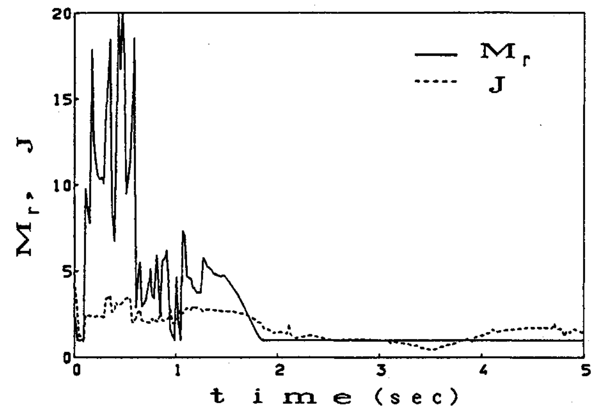


Fig. 7 Time histories of M_r and J in the failure case b.

in the normal case. The time responses of the control surfaces show that, since the failure causes asymmetric motion with respect to X-Z plane, the left and the right surfaces move asymmetrically, which implies that the functional redundancy is utilized.

3) This is a more demanding case. Figures 4 show that the transient responses of angle of attack and pitch angle are acceptable and the tracking errors at the steady state are nearly zero, which means that the aircraft almost makes a level flight. However, whereas the responses of sideward speed and roll angle tend to be damped, the remaining surfaces are unable to make the outputs track their references. This is because the rudder, which is a dominant effector of yaw control, is stuck. It implies that the failure of rudder jam has a crucial effect on lateral-directional control and that the other available surfaces cannot make up for its function well. To accommodate this failure, the aircraft is required to have redundant yaw effectors such as split rudders (upper and lower), double vertical tails, or vertical canards.

4) From the time responses of the outputs in Figs. 5, it is seen that the motions cannot be controlled without restructure in this case. Compared with case 3, there are considerable effects of restructure on safety of the aircraft.

To examine the convergence property of the output tracking error ϵ and the identification error y, the time histories of y and ϵ with respect to sideward speed for the failure case a are plotted in Fig. 6. In the figure, ϵ is scaled by 50. From Eq. (20), $y = \epsilon'/(s+5)^2$, for $y = v$. Although the relation between ϵ and ϵ' is not clear, the figure shows that y follows ϵ with some delay, and that after ϵ disappears, y does exponentially. This result implies that there is a similar relation between y and ϵ to that between y and ϵ'. It remains to say that if actuator dynamics can be neglected, a definite relation like Eq. (20) exists between y and ϵ, and that $y \to 0$ is guaranteed if only $\epsilon \to 0$.

Finally, it is evaluated how much the loss of control power that could be used if the generic inputs were not modified in saturation of the surfaces would be. Figure 7 displays the time histories of M_r given by Eq. (25) and the value,

$$J = \sum_i |\delta_i| / |\delta_{i\max}| \tag{39}$$

where ($i = hR, aL, aR, cL, cR$) for the failure case b. From the definitions of M_r and J, it is seen that, if $M_r = 1$, then the generic inputs are not modified, hence, no real input is modified, and that, if $M_r > 1$, then $(5 \geq) J \geq 1$. The larger M_r is, the more inputs reduce. The worst is the case where M_r is very large and J is nearly equal to 1, which means that one real input exceeds the limit too much, compared with others, so that they are greatly decreased and the available control power is discarded. Even in such a case, if J is nearly equal to 5, the loss can be regarded small. For example, in Fig. 7, whereas M_r is large from the time 0.1 to 1.8 s, J takes a value from 2 to 3 during the same period. It may seem to be a considerable loss, but in the rest of the simulation time, $M_r = 1$, and in addition, unless the generic inputs are not modified, the time responses of the outputs diverged probably due to the wrong estimated parameters. Therefore, at the expense of some loss of the control power, the modification is indispensable.

V. Summary and Conclusions

The proposed RFCS is based on the feedback linearization control method, taking actuator dynamics into account. The control distributor and generic inputs are employed to reduce the number of inputs to that of outputs. The other effect of CD is to reduce the number of parameters to be identified, which will alleviate the burden to the computer. The modification of generic inputs is introduced to remove the contradiction between generic inputs and real saturated inputs. Although the modification may reduce the control power that could be used, it is indispensable because, if the generic inputs are not modified, in some failure cases, the time responses of the aircraft diverge.

Two failure cases are examined by computer simulation. The RFCS works well, except for the fact that the lateral-directional control performance is poor in the failure case b, probably because the rudder, the main yaw effector, is stuck. This fact implies that, although the RFCS takes advantage of the functional redundancy of the effectors, the dominant one with respect to at least pitch, yaw, roll, and thrust (power) should be redundant.

Since the RFCS identifies failures by estimating parameters of the aircraft equations of motion, it has the potential to accommodate all failures whose effects appear in the parameters of the equations of motion, including simultaneous failures and those changing open-loop stability, as long as the sufficient control power remains. However, the problems such as computation delay and robustness must be solved.

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